ANÁLISIS MATEMÁTICO II – FINAL RESUELTO - 17/12/13

Parte práctica

3.- Hallar y clasificar los puntos críticos de la función $z(x,y)$, dada en forma implícita:

$$2x^2+2y^2+z^2+8xz-z+8=0$$

4.- Verificar el teorema de la divergencia de Gauss, si $\mathbf{F}(x,y,z)=\left(x+y^2,-2x,2yz\right)$, siendo S la superficie cerrada limitada por: $z=x^2+y^2$; $0\le z\le 4$.

5.- Sea $F(x,y,z)=xy+a\,yz+zx$. Determinar el valor de la constante a para el cual la derivada de la función en el punto $(1,3,2)$ hacia el punto $(5,15,5)$ resulte ser $\dfrac{81}{26}$

3.- $f(x,y,z)=2x^2+2y^2+z^2+8xz-z+8=0$

$$\frac{\partial z}{\partial x}=-\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial z}}=-\frac{4x+8z}{2z+8x-1}=0\rightarrow z=-\frac{1}{2}x \qquad reempl.\ en\ (1)\rightarrow x_1=\frac{16}{7}\ \ x_2=-2$$

$$\frac{\partial z}{\partial y}=-\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial z}}=-\frac{4y}{2z+8x-1}=0\rightarrow y=0$$

$$P_1=\left(\frac{16}{7},0,-\frac{8}{7}\right)\quad P_2=(-2,0,1)$$

Clasificación:

$$\frac{\partial^2 z}{\partial x^2}=-\frac{\left(4+8\frac{\partial z}{\partial x}\right)(2z+8x-1)-(4x+8z)\left(2\frac{\partial z}{\partial x}+8\right)}{(2z+8x-1)^2}$$

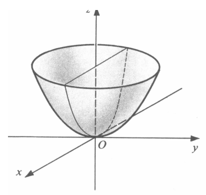
$$\frac{\partial^2 z}{\partial y\partial x}=-\frac{8\frac{\partial z}{\partial y}(2z+8x-1)-(4x+8z)2\frac{\partial z}{\partial y}}{(2z+8x-1)^2}$$

$$\frac{\partial^2 z}{\partial y^2}=-\frac{4(2z+8x-1)-4y2\frac{\partial z}{\partial y}}{(2z+8x-1)^2}$$

$$\left.\frac{\partial^2 z}{\partial x^2}\right|_{P_1}<0\quad \left.\frac{\partial^2 z}{\partial y^2}\right|_{P_1}<0\quad \left.\frac{\partial^2 z}{\partial y\partial x}\right|_{P_1}=0\rightarrow H(P_1)>0\ y\ \left.\frac{\partial^2 z}{\partial x^2}\right|_{P_1}<0\rightarrow \max$$

$$\left.\frac{\partial^2 z}{\partial x^2}\right|_{P_2}>0\quad \left.\frac{\partial^2 z}{\partial y^2}\right|_{P_2}>0\quad \left.\frac{\partial^2 z}{\partial y\partial x}\right|_{P_2}=0\rightarrow H(P_2)>0\ y\ \left.\frac{\partial^2 z}{\partial x^2}\right|_{P_2}>0\rightarrow \min$$

4.- La superficie es un paraboloide circular sobre el plano *xy*, cerrado con un circulo de radio 2 a la altura de $z=4$.



a) Integral triple

Empleamos coordenadas cilíndricas

$x = r\cos t ; y = r\,sen t ; z = z$

$0 \le z \le 4 ; 0 \le t \le 2\pi ; 0 \le r \le \sqrt{z}$

$div\,F = 1 + 2y = 1 + 2r\,sen t \quad |J| = r$

$\int_0^4 \int_0^{\sqrt{z}} \int_0^{2\pi} \left(r + 2r^2\, sen t \right) dt\ dr\ dz = 8\pi$

b) En la integral de superficie tenemos dos casos, la tapa circular y la superficie del paraboloide.

<u>Tapa circular</u>

$x^2 + y^2 = 4 \quad z = 4$

$r(u,v) = (u\cos v, u\,sen v, 4)$

$T_u = \frac{\partial r}{\partial u} = (\cos v, sen v, 0)$

$T_v = \frac{\partial r}{\partial v} = (-u\,sen v, u\cos v, 0)$

$N = T_u \times T_v = (0,0,r)$

$F[r(u,v)] = \left(u\cos v + u^2\, sen^2 v, -u\cos v, 8u\,sen v \right)$

$F.N = 8u\,sen v$

$\iint_{S_1} F.N\,dS = \int_0^2 \int_0^{2\pi} 8u\,sen v\ dv\,du = 0$

<u>Paraboloide:</u>

$z = x^2 + y^2 \quad 0 \le z \le 4$

$r(t,z) = \left(\sqrt{z}\cos t, \sqrt{z}\ sen t, z \right) \quad 0 \le t \le 2\pi \quad 0 \le z \le 4$

$T_t = \left(-\sqrt{z}\ sen t, \sqrt{z}\cos t, 0 \right)$

$T_z = \left(\frac{1}{2\sqrt{z}}\cos t, \frac{1}{2\sqrt{z}}\,sen t, 1 \right)$

$N = T_t \times T_z = \left(\sqrt{z}\cos t, \sqrt{z}\,sen t, -\frac{1}{2} \right)$

$F[r(t,z)].N = z\cos^2 t + z\sqrt{z}\ sen^2 t\cos t - 2z\cos t\,sen t - z\sqrt{z}\ sen t$

$\iint_{S_2} F.N\,dS = \int_0^4 \int_0^{2\pi} \left(z\cos^2 t + z\sqrt{z}\ sen^2 t\cos t - 2z\cos t\,sen t - z\sqrt{z}\ sen t \right) dt\,dz = 8\pi$

5.-

$\vec{v} = (4,12,3) \rightarrow \|v\| = 13 \rightarrow \vec{v} = \left(\frac{4}{13}, \frac{12}{13}, \frac{3}{13} \right)$

$\nabla f = (y + z, x + az, ay + x)\big|_P = (5, 1 + 2a, 3a + 1)$

$\nabla f.\vec{v} = \frac{81}{26} \rightarrow 20 + 12 + 24a + 9a + 3 = \frac{81}{2} \rightarrow a = \frac{1}{6}$