SEGUNDO RECUPERATORIO ANÁLISIS II - 6/7/2013

1.- Calcular $J(f \circ g)_{(1,-1,1)}$, siendo $g: \mathbb{R}^3 \to \mathbb{R}^2 \,/\, g(x,y,z) = \left(\text{sen}\,(xy+z), \left(1+x^2\right)^{yz}\right)$ y $f: \mathbb{R}^2 \to \mathbb{R}^2 \,/\, f(u,v) = \left(u+e^v, v-e^u\right)$

2.- Hallar $\iint_S \mathbf{F}.\mathbf{N}\, dS$, si $\mathbf{F}(x,y,z) = (x,y,z)$ y S es la superficie cerrada formada por $x^2+y^2=9$, $z=0$, $z=5$.

3.- Calcular los extremos de $f(x,y,z) = x-2y+2z$ con la condición $x^2+y^2+z^2=9$.

4.- Evaluar la integral de línea $\int_C 2xyz\,dx + \left(x^2 z+1\right)dy + \left(x^2 y-1\right)dz$, donde C es la curva $x-y^2+z=1$, desde el punto $(1,1,1)$ al (1,2,4).

Resolución

1.-

$$J(f \circ g)(1,-1,1)_{2\times3} = Jf\left[g(1.-1,1)\right]_{2\times2}\; Jg(1.-1.1)_{2\times3}$$

$$g(1.-1.1) = \left(0, \frac{1}{2}\right)$$

$$Jf = \begin{pmatrix} 1 & e^v \\ -e^u & 1 \end{pmatrix}_{(0,\frac{1}{2})} = \begin{pmatrix} 1 & e^{\frac{1}{2}} \\ -1 & 1 \end{pmatrix}$$

$$Jg = \begin{pmatrix} y\cos(xy+z) & x\cos(xy+z) & \cos(xy+z) \\ 2xyz(1+x^2)^{yz-1} & z(1+x^2)^{yz}\ln(1+x^2) & y(1+x^2)^{yz}\ln(1+x^2) \end{pmatrix}_{(1,-1,1)} =$$

$$= \begin{pmatrix} -1 & 1 & 1 \\ -\frac{1}{2} & \frac{1}{2}\ln 2 & -\frac{1}{2}\ln 2 \end{pmatrix}$$

$$J(f \circ g)(1,-1,1) = \begin{pmatrix} -1-\frac{1}{2}e^{\frac{1}{2}} & 1+\frac{1}{2}e^{\frac{1}{2}}\ln 2 & 1-\frac{1}{2}e^{\frac{1}{2}}\ln 2 \\ \frac{1}{2} & -1+\frac{1}{2}\ln 2 & -1-\frac{1}{2}\ln 2 \end{pmatrix}$$

2.-

$$div\, F = 3$$

$$\begin{cases} x = r\cos t & 0 \le r \le 3 \\ y = r\, sen t & 0 \le t \le 2\pi \qquad |J| = r \\ z = z & 0 \le z \le 5 \end{cases}$$

$$\int_0^3 \int_0^{2\pi} \int_0^5 3r\, dz\, dt\, dr = 225\pi$$

3.-

$$L(x,y,z,\lambda) = x-2y+2z+\lambda(x^2+y^2+z^2-9)$$

$$L_x = 1 + 2\lambda x = 0 \rightarrow x = \frac{-1}{2\lambda} \quad \lambda \neq 0$$

$$L_y = -2 + 2\lambda y = 0 \rightarrow y = \frac{1}{\lambda}$$

$$L_z = 2 + 2\lambda z = 0 \rightarrow z = -\frac{1}{\lambda}$$

$$\therefore \frac{1}{4\lambda^2} + \frac{1}{\lambda^2} + \frac{1}{\lambda^2} = 9 \rightarrow \lambda = \pm\frac{1}{2}$$

$$Si \ \lambda = \frac{1}{2} \ P_1 = (-1,2,-2) \quad Si \ \lambda = -\frac{1}{2} \ P_2 = (1,-2,2)$$

$$\tilde{H} = \begin{pmatrix} 0 & 2x & 2y & 2z \\ 2x & 2\lambda & 0 & 0 \\ 2y & 0 & 2\lambda & 0 \\ 2z & 0 & 0 & 2\lambda \end{pmatrix} \rightarrow P_1 \ \min \qquad P_2 \ \max$$

4.-

$$rot\,\mathbf{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz & x^2z+1 & x^2y-1 \end{vmatrix} = (0,0,0) \ \mathbf{F} \text{ es un campo conservativo.}$$

Obtenemos la función potencial.

$$f(x,y,z) = \int 2xyz\,dx + g(y,z) = x^2yz + g(y,z)$$

$$\frac{\partial f}{\partial y} = x^2z + \frac{\partial g}{\partial y} = F_2 = x^2z + 1 \rightarrow \frac{\partial g}{\partial y} = 1 \rightarrow g(y,z) = \int 1dy + h(z)$$

$$f(x,y,z) = x^2yz + y + h(z) \rightarrow \frac{\partial f}{\partial z} = x^2y + h'(z) = F_3 = x^2y - 1 \rightarrow$$

$$\rightarrow h'(z) = -1 \rightarrow h(z) = -z + C$$

$$f(x,y,z) = x^2yz + y - z + C \rightarrow W = f(1,2,4) - f(1,1,1) = 6 - 1 = 5$$