Análisis Matemático II – Segundo recuperatorio 08/7/2014

1.- Hallar y clasificar todos los puntos críticos de $f(x,y)=x^3+y^3+3xy^2-4(x+y)$.

2.- Sea $f(x,y,z)=3xy^2+4yz-azx^2$, determinar los valores de *a* para que la derivada de *f* en $P=(2,1,3)$ hacia $Q=(5,5,15)$ resulte ser $\frac{29}{221}a^2$.

3.- Calcular $\int_{-2}^{0}\left[\int_{-x}^{\sqrt{8-x^2}}\frac{dy\,dx}{\sqrt{1+x^2+y^2}}\right]+\int_{0}^{\sqrt{8}}\left[\int_{0}^{\sqrt{8-x^2}}\frac{dy\,dx}{\sqrt{1+x^2+y^2}}\right]$

4.- Comprobar el teorema de Stokes si $\mathbf{F}(x,y,z)=(-y,x,1)$ siendo *S*: $x^2+y^2+z^2=1;\ z\geq 0$.

Soluciones

1.-

$$\frac{\partial f}{\partial x}=3x^2+3y^2-4=0 \quad (1)$$

$$\frac{\partial f}{\partial y}=3y^2+6xy-4=0 \quad (2)\rightarrow 3y^2=4-6xy\rightarrow en\ (1)\quad 3x^2+4-6xy-4=0\rightarrow 3x(x-2y)=0\rightarrow x=0\ o\ x=2y$$

$$Si\ x=0\ en\ (2)\rightarrow y^2=\frac{4}{3}\rightarrow y=\pm\frac{2}{\sqrt{3}}\rightarrow P_1=\left(0,\frac{2}{\sqrt{3}}\right)\quad P_2=\left(0,-\frac{2}{\sqrt{3}}\right)$$

$$Si\ x=2y\ en\ (2)\rightarrow y^2=\frac{4}{15}\rightarrow y=\pm\frac{2}{\sqrt{15}}\rightarrow P_3=\left(\frac{4}{\sqrt{15}},\frac{2}{\sqrt{15}}\right)\quad P_4=\left(-\frac{4}{\sqrt{15}},-\frac{2}{\sqrt{15}}\right)$$

$$H=\begin{pmatrix}6x & 6y\\ 6y & 6y+6x\end{pmatrix}$$

$$H(P_1)<0\rightarrow Pto\,silla;\ H(P_2)<0\rightarrow Pto\,silla;\ H(P_3)>0\ \ \frac{\partial^2 f}{\partial x^2}>0\rightarrow \min rel;\ H(P_4)>0\ \ \frac{\partial^2 f}{\partial x^2}<0\rightarrow \max rel$$

2.-

$$v=Q-P=(3,4,12)\rightarrow\|v\|=13\rightarrow\breve{v}=\left(\frac{3}{13},\frac{4}{13},\frac{12}{13}\right)$$

$$\nabla f(x,y,z)=\left(3y^2-2azx,6xy+4z,4y-ax^2\right)\rightarrow\nabla f(2,1,3)=(3-12a,24,4-4a)$$

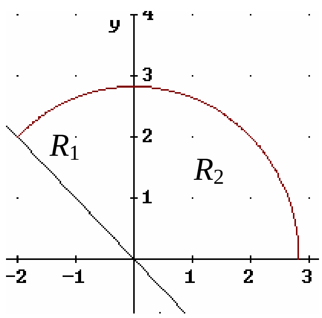
$$\nabla f(2,1,3).\breve{v}=\frac{1}{13}(9-36a+96+48-48a)=\frac{29}{221}a^2\rightarrow 29a^2+1428a-2601=0\rightarrow a_1=\frac{51}{29};\ a_2=-51$$

3.-

$$R_1=\begin{cases}-2\leq x\leq 0\\ -x\leq y\leq\sqrt{8-x^2}\end{cases}\qquad R_2=\begin{cases}0\leq x\leq\sqrt{8}\\ 0\leq y\leq\sqrt{8-x^2}\end{cases}$$

En polares

$$R=\begin{cases}0\leq r\leq\sqrt{8}\\ 0\leq\theta\leq\frac{3}{4}\pi\end{cases}\qquad |J|=r\qquad f(r,\theta)=\frac{1}{\sqrt{1+r^2}}\rightarrow I=\int_0^{3/4\pi}\int_0^{\sqrt{8}}\frac{r}{\sqrt{1+r^2}}\,dr\,d\theta=\frac{3}{2}\pi$$



4.-

$a) \iint_S \mathbf{rotF}.\mathbf{N}\, dS$

$r(u,v) = (\cos u \cos v, \operatorname{sen} u \cos v, \operatorname{sen} \mathrm{v}) \to 0 \le u \le 2\pi \ ; \ 0 \le v \le \dfrac{\pi}{2}$

$T_u = (-\operatorname{sen} u \cos v . \cos u \cos v, 0)$

$T_v = (-\cos u \operatorname{sen} v, -\operatorname{sen} u \operatorname{sen} v, \cos v)$

$N = T_u \times T_v = (\cos u \cos^2 v, \operatorname{sen} u \cos^2 v, \operatorname{sen} v \cos v)$

$rotF = (0,0,2)$

$rotF.N = 2 \operatorname{sen} v \cos v$

$$\int_0^{2\pi} \int_0^{\pi/2} 2 \operatorname{sen} v \cos v \, dv \, du \quad = 2\pi$$

$b) \oint_C F.dr$

$r(t) = (\cos t, \operatorname{sen} t, 0) \to r'(t) = (-\operatorname{sen} T, \cos t, 0) \to F[r(t)] = (-\operatorname{sen} t, \cos t, 1) \to F.r' = \operatorname{sen}^2 t + \cos^2 t = 1$

$$\int_0^{2\pi} \quad dt = 2\pi$$